CALCULATING THE HEAT TRANSFER IN A TURBULENT BOUNDARY LAYER OF VISCOUS FLUID WITH A HIGH PRANDTL NUMBER

M. E. Podol'skii

As is well known, calculations of the heat transfer in a turbulent boundary layer yield too low values for the thermal flux when based on the two-layer or the three-layer model. It appears that the two-layer model will yield entirely satisfactory results at a high Prandtl number, if the additional heat transfer due to turbulent fluctuations in the laminar sublayer is taken into account. The formulas here are also quite simple and convenient for use on the computer.

With turbulent heat transfer taken into account, the formula for the thermal conductivity λ_σ of the laminar sublayer becomes

$$\lambda_\sigma = \lambda + \rho c v_t'. \tag{1}$$

Here v_t' denotes the coefficient of turbulent viscosity which, according to the Landau–Levich hypothesis, will be considered proportional to the fourth power of the distance from the wall. The thickness of the laminar sublayer will be denoted by δ_l and the dynamic velocity by v^*. We then have for v_t':

$$v_t' = n\eta^4 v. \tag{2}$$

In this formula

$$n = \frac{k}{\eta_l^3}; \quad \eta = \frac{yv^*}{v}; \quad \eta_l = \frac{\delta_l v^*}{v};$$

and k is the proportionality factor between mixing length and distance from the wall.

In combination with (1), formula (2) yields the following expression for the Stanton number (based on the velocity of the oncoming stream U and the temperature drop $T_0 - T_f$ across the boundary layer):

$$St = \frac{\dfrac{c_f}{2}}{1 + \sqrt{\dfrac{c_f}{2}} \, [\psi\,(Pr)\,Pr^{3/4} - \eta_l]}; \quad c_f = \frac{2\tau}{\rho U^2}.$$

Here τ denotes the frictional stress at the wall and $\psi(Pr)$ is a function of the Prandtl number, quasiconstant when $Pr > 1$ ($\psi \approx 8.4$). For pipes, where the Stanton number is usually based on the mean velocity v_m and the mean-over-the-flow temperature $T_0 - T_m$, we have

$$St_m = \frac{(\varphi_m/\vartheta_m)\,(\zeta/8)}{1 + \varphi_m \sqrt{\zeta/8} \, [\psi\,(Pr)\,Pr^{3/4} - \eta_l]},$$

$$\varphi_m = \frac{u_m}{U}; \quad \vartheta_m = \frac{T_0 - T_m}{T_0 - T_f}; \quad \zeta = \frac{8\tau}{\rho u_m^2}. \tag{3}$$

Calculations by formula (3) are in satisfactory agreement with experimental data and with values obtained by Snegova with the van Dreest formula for the mixing length.

ENERGY AND MATERIAL TRANSFER PROCESSES INCLUDING THE EFFECT OF PRESSURE AND CHEMICAL CONVERSION IN NATURAL HYDROTHERMAL SYSTEMS

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UDC 532.72:536.242

Natural processes which occur during the interaction between hydrothermal solutions (or magnetic melts) and rocks deep inside the earth crust are governed by high rates of heat and mass transfer [4] between the liquid and the solid phase [2]. The development of temperature and humidity fields as a result of hydrothermal interactions is accompanied by a transport of chemical components within the geological system.

The rates of heat and mass transfer determine the system kinetics and the character of the chemical reactions. Chemical conversions under natural conditions are accompanied by stronger and weaker thermal effects representing internal heat sources in the hydrothermal process. Because of the long development and formation period in the case of geological systems, such rather weak thermal effects compound with time and may deform both the temperature and the humidity fields.

For a quantitative study of the general laws govering the energy and the material transfer in hydrothermal systems, it has become necessary to develop models simulating nearly actual conditions.

The mathematical model proposed here describes, to a closer approximation than earlier models [3], the natural hydrothermal process including both the dynamics and the thermal effects of chemical conversions, which, in the final analysis, represent crystallization, recrystallization, dissolution of minerals, and metasomatic transformations of matrix rocks.

The mathematical model of the process comprises a system of parabolic partial differential equations describing the interdependent heat and mass transfers, the diffusion of chemical components, and the pressure changes in the interstitial solution of the (hydrothermal)solution—rocks system.

A method is outlined for synthesizing the mathematical model on a digital computer, under constraints approaching the natural ones.

The problem is solved by the two-dimensional finite-differences scheme [1] and the Runge—Kutta method [6], both needing to be evaluated for accuracy and stability of the solution to this system of equations. An error and convergence analysis of the solution shows that this procedure can be used successfully for solving not only individual equations [5] but also multidimensional systems of functionally coupled differential equations.

For this reason, the results obtained by synthesis of the mathematical model describe accurately enough the general laws which govern the natural hydrothermal process and they are useful for studying the mechanism by which a mineralizing solution is transported, the formation of ore deposits, and the metamorphism of rocks.

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MASS TRANSFER DURING A TRANSVERSE FLOW
AROUND A CIRCULAR CYLINDER

M. Kh. Kishinevskii and A. A. Mosyak   UDC 536.24:532.526+532.72

Most test data on heat and mass transfer, at a high Prandtl number, between a circular cylinder and a stream flowing perpendicularly to its axis have so far been obtained for a Reynolds number and a Prandtl number not exceeding Re = 12,000 and Pr = 2800 respectively [1-5].

The authors have used the rotation method in an attempt to extend the range of both numbers. A stationary specimen was placed in a stream of liquid filling an annular channel of rectangular cross section and revolving around the vertical axis. The lack of a free liquid surface in the revolving channel helped to reduce the perturbations due to interference between the specimen and the channel walls or bottom; this, in turn, made it possible to attain a high Reynolds number during the flow around the specimen. Two rotary devices of different sizes were used in the experiment. On one of them the transfer coefficients were determined from the loss of specimen mass, on the other they were determined from the change in benzoic acid concentration in the solution. A series of mass transfer tests was also performed in a stationary channel of rectangular cross section.

Test specimens 0.0351 and 0.0085 m in diameter with a 0.012-0.020 m height of the dissolving segment had been prepared by molding benzoic acid powder under 1500 bars pressure. Annular extensions were fastened to the specimens and only the center portion of benzoic acid with a grade-9 surface purity was allowed to dissolve.

The solvents were distilled water and aqueous 22.8-67.5% (weight) glycerine solutions with the concentration of benzoic acid varying from zero to 50-70% of the saturation level at 25 ± 0.2°C.

The stream velocity at the critical point in front was calculated from the difference between total and static pressure at the stagnation point.

The values of local mass transfer coefficients were determined from the change in the transverse dimensions of the dissolving 0.0351 m cylindrical segments. It could be established, in this way, that the dissolution had occurred uniformly along the cylinder height and symmetrically with respect to its axis. The local mass transfer coefficients in the vicinity of the stagnation point determined thus were on the average 5% lower than those calculated by the theoretical formula for $Pr \rightarrow \infty$ [6]:

$$Nu = 1.332 \, Re^{0.50} \, Pr^{0.33} \tag{1}$$

The mean-over-the-diameter mass transfer coefficients within the ranges Re = 4300-410,000 and Pr = 930-200,000 correlate by the equation

$$Nu = 0.082 \, Re^{0.73} \, Pr^{0.32}. \tag{2}$$

For determining the Prandtl number, the velocity of the oncoming stream in formula (2) was multiplied by a factor accounting for channel clogging and calculated by the method in [7].

NOTATION

Pr   is the Prandtl diffusion number;
Re   is the Reynolds number;
Nu   is the Nusselt diffusion number.

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EXPERIMENTAL STUDY OF HEAT TRANSFER IN MESHED MATRICES*

E. I. Mikhulin and Yu. A. Shevich

Results are presented of an experimental study concerning the heat transfer during a forced flow of air through fine-mesh matrices (d_{wire} = 0.03-0.08 mm, d_{cell} = 0.04-0.112 mm) of domestic manufacture.

The tests were performed under steady conditions with a continuous heat generation in the metal by means of electric current.

Matrices were tested with various numbers of meshes per packet, from one to several hundred, i. e., over a wide range of the H/D_e ratio (H denoting the packet length in the direction of air flow and D_e denoting the equivalent diameter).

It has been established that, as the ratio H/D_e is made larger, the rate of heat transfer in meshed matrices decreases while the heat transfer at a single mesh is the same as that at a cylinder in a transverse stream, but the heat transfer in a packet with many meshes does, to some extend, approach that in a solid channel.

The effect of the H/D_e ratio and of other parameters explains the wide discrepancy between known data on heat transfer in meshed matrices.

Criterial equations are proposed for describing the heat transfer in fine-mesh matrices with varying relative length and within the range of Reynolds number Re = 10-500.

EVALUATING THE EFFECTS OF UNSTEADY EVAPORATION AND CONDENSATION †

I. Ya. Kolesnik

The solution to the equations describing unsteady condensate buildup and evaporation processes with a movable interphase boundary is written in the form

$$r^2 = 1 + 2\tau + \frac{\sqrt{2}}{\sqrt{\pi}} [(1+2\tau) \operatorname{arctg} \sqrt{2\tau} + \sqrt{2\tau}] \, v + 2 \left[\frac{d}{d\tau} \left(\frac{1}{\sqrt{\pi\tau}} * r_0^2 r_1 \right) - (1+\tau) \right] v^2 + \ldots, \tag{1}$$

$$r^2 = 1 - 2\tau - \frac{\sqrt{2}}{\sqrt{\pi}} \left[\sqrt{2\tau} + (1-2\tau) \ln \frac{1+\sqrt{2\tau}}{\sqrt{1-2\tau}} \right] v + 2 \left[\frac{d}{d\tau} \left(\frac{1}{\sqrt{\pi\tau}} * r_0^2 r_1 \right) + (1-\tau) \right] v^2 - \ldots \tag{2}$$

With the aid of expansions (1) and (2), the following formulas are derived, to the first approximation with respect to the small parameter ν:

for the rate of condensate buildup

$$J^{**}=J\left\{1+\nu\varphi\left(\tau\right)\right\}, \tag{3}$$

for the evaporation rate

$$J^{**}=J\left\{1+\nu\varphi_1\left(\tau\right)\right\},$$

where

$$\varphi\left(\tau\right)=\frac{3}{\sqrt{2\pi}}\operatorname{arctg}\sqrt{2\tau}+\frac{1+3\tau}{\sqrt{\pi\tau}\left(1+2\tau\right)};$$

$$\varphi_1\left(\tau\right)=\frac{3}{\sqrt{2\pi}}\ln\frac{\sqrt{1-2\tau}}{1+\sqrt{2\tau}}+\frac{1-3\tau}{\sqrt{\pi\tau}\left(1-2\tau\right)};$$

and J is the rate of quasisteady evaporation or condensate buildup.

According to relation (3), even at $\tau \geq 1$ the correction to the quasisteady flow rate during condensate buildup is almost twice as large as the correction suggested by an earlier formula by other authors. As $\tau \to 0$, the difference between both corrections increases infinitely. The effect of unsteadiness on the process rate at some fixed τ is determined entirely by the value of the small dimensionless parameter ν, i.e., by the temperature and the supersaturation.

In the case of evaporation, the correction changes sign. It is positive at τ within the interval $0 \leq \tau < \tau_1$ ($\tau_1 \approx 0.26$) and is negative at $\tau > \tau_1$, while, according to the earlier formula, the correction is always negative, i.e., the actual rate is always lower than the quasisteady rate. The difference between both corrections becomes appreciable, in this case, when $\tau \to 0$ and at a time close to complete evaporation of a condensate.

The formulas derived here are useful for studying and evaluating all basic effects related to unsteadiness resulting from both a discontinuous concentration field at the initial instant of time and a movable interphase boundary, they also indicate the limitations on the applicability of the quasisteady approximation in evaporation and condensate buildup theory.

THREE-DIMENSIONAL THERMAL SHOCK OF FINITE
DURATION IN A TRANSMITTENT PLATE

V. G. Andreev and P. I. Ulyakov   UDC 539.3:536

The presence of high temperature gradients during a short-time thermal shock requires that the analysis be based on the hyperbolic equation of heat conduction, which accounts for a finite velocity of heat propagation. Heat is transmitted through dielectric lattices principally by the conduction mechanism, and the velocity of heat propagation is here equal to the velocity of sound in the given medium. A propagation of temperature and stress perturbations at the same velocity indicates that both travel on the same wave. When the initial conditions are of a discrete kind (momentary shock), there occurs a jump in pressure as well as in temperature and in density, with the result that the equations of thermoelasticity become inapplicable as far as determining the parameters of the given medium under a discontinuity.

In actual processes a thermal shock is of a finite duration and the stresses build up continuously behind the wave front. The dynamic problem of thermoelasticity is solved in this article by means of the

Laplace transformation for a three-dimensional thermal shock of finite duration. Expressions are derived for the temperatures and the stresses, while the problem is solved simultaneously in the parabolic approximation. The quasistatic state of stress represents a special case (at $c_0 \to \infty$).

By introducing into the problem a velocity of heat propagation equal to the velocity of sound, it has become possible to eliminate the physically absurd appearance of stresses prior to the arrival of a wave at a given point.

An analysis of the solution shows that the amplitude of the temperature front decays exponentially with time. It subsequently does not affect the propagation of stress waves. A quasistatic stress field is established behind the front of an elastic wave.

CONCERNING THE VALUE OF THE CONSTANT-VOLUME
SPECIFIC HEAT AT THE CRITICAL POINT

Kh. I. Amirkhanov, B. G. Alibekov,
and G. V. Stepanov

On the basis of available test data pertaining to the specific heat C_V of water, argon, xenon, nitrogen, and oxygen, this constant-volume specific heat is shown here to have a finite value at the critical point. There is evidently no justification for letting it become infinite at that point, as some other authors have done by extrapolating empirical formulas (the logarithmic or the power-law expression) to the critical point. It is shown, instead, that these formulas describe with equal accuracy the test results pertaining to the critical isochor and to adjacent isochors, and that, therefore, the results of such an extrapolation would violate the laws of thermodynamics.

On the basis of a numerical evaluation of straight test data pertaining to the constant-volume specific heat of the substances examined most thoroughly, namely water and argon, another empirical formula is given to describe the trend of C_V along the critical isochor, with a standard deviation smaller than that along the logarithmic curve. At the same time, the formula proposed here by the authors yields, by extrapolation, finite values for the constant-volume specific heat at the critical point.

Graphs are shown also for the other substances listed here, indicating a closer agreement between test data and the proposed formula of the

$$C_v = a \exp \left[-b \, | \, T_k - T |^{0,1} \right].$$

kind. The value of coefficients a and b for water and argon are determined by the method of least squares (for argon $a = 556.2$ J/mole·deg C and b = 2.487; for water $a = 12.62$ cal/g·deg C and b = 1.403 at $T < T_{cr}$, $a = 7.27$ cal/g·deg C and b = 1.668 at $T > T_{cr}$). It is also shown that the interpolation interval based on these formulas covers a wider temperature range.

Furthermore, test data on C_V along the saturation line are also evaluated and a subsequent analysis shows that at the critical point the specific heat C_V remains finite on the two-phase side as well as on the single-phase side, with a finite jump at that point.

EQUIVALENT DIAMETER AND MINIMUM NUSSELT NUMBER FOR AN ELLIPSOIDAL PARTICLE AT A REYNOLDS NUMBER Re = 0*

B. M. Abramzon

Calculating the convective heat transfer at a particle of irregular shape reduces to the problem of finding its equivalent diameter d_e, i.e., the diameter of a sphere at which the heat transfer rate under given conditions is the same as at that particle. Three approximate definitions have been proposed in the technical literature for this equivalent diameter:

1. the mean-volume diameter $d_V = \sqrt[3]{6V/\pi} = d_e$,

2. the mean-surface diameter $d_S = \sqrt{S/\pi} = d_e$, and

3. the diameter of a sphere whose volume to surface ratio is the same as that of the given particle: $d_e = 6V/S = d_V/f$ (with V denoting the particle volume, S denoting the particle surface, and the asphericity factor $f = S/\pi d_V^2$).

A solution is given here to the problem of heat and mass transfer rates at an ellipsoid immersed in an infinitely large stationary medium.
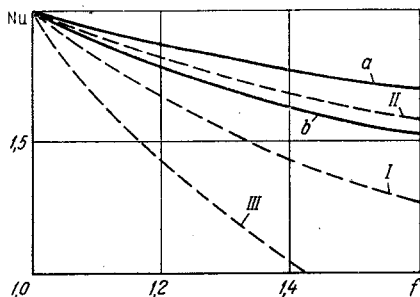

Fig. 1

The exactly calculated Nusselt number is shown in Fig. 1 for a prolate ellipsoid (curve a) and an oblate ellipsoid (curve b) of revolution, as a function of the asphericity factor. For comparison, curves I, II, and III representing respectively the three approximate definitions of the equivalent diameter have also been plotted here (the Nusselt number is based on the mean-volume diameter of a particle).

Thus, most accurate is the method in which the mean-volume diameter of a particle is defined as its equivalent diameter for approximate calculations.

DIMENSIONLESS PRESSURE HEAD VERSUS FLOW RATE CHARACTERISTICS OF NON-NEWTONIAN FLUIDS IN THE CHANNEL OF A COMPOUND-DISPLACEMENT SCREW PUMP†

V. I. Yankov, L. M. Beder,
S. A. Bostandzhiyan, V. I. Boyarchenko,
and V. V. Kiselev

Isothermal steady flow of an anomalously viscous fluid with a power-law rheological characteristic is analyzed, under conditions in the channel of a compound-displacement screw pump. Formulas are derived for the maximum axial and transverse pressure gradient. Knowing these values, one can introduce dimensionless pressure gradients and flow rates for plotting the pressure head versus flow rate characteristics of screw pumps with various pitch angles and fluids with various viscosity anomalies — characteristics which are invariant with respect to fluid viscosity, tangential velocity, and screw dimensions.

With the aid of these characteristics and auxiliary graphs, one can calculate the pressure and the power requirements of a screw pump without the use of computers.

ADIABATIC FLOW OF A NON-NEWTONIAN FLUID IN THE CHANNEL OF A COMPOUND-DISPLACEMENT SCREW PUMP *

V. I. Yankov, L. M. Beder, S. A. Bostandzhiyan, and V. I. Boyarchenko

Adiabatic flow of a fluid with a power-law rheological characteristic in the channel of a screw pump or any other screw machine is analyzed, taking into account the circulatory flow in such a channel.

It is assumed that no heat transfer occurs with the ambient medium and that the fluid temperature varies only along the channel, as a result of dissipative heating, but remains constant in two other measurements and, on this basis, convenient engineering formulas are derived for the temperature, the pressure drop, and the power requirement. An equation is obtained for the pump efficiency, it is also shown how the pump efficiency depends on the viscosity anomaly and on the screw pitch angle.

ACCOUNTING FOR TWO-DIMENSIONAL FLOW IN A THROTTLE TYPE DUST FLOW METER †

Yu. V. Rzheznikov and É. V. Lifshits

The authors analyze the performance of a throttle device (Fig. 1), a flat channel (width H_0) with an axially mounted expeller (width H) consisting of a small head and a sufficiently long plate (length l) behind it, in a dusty stream. The head and the plate are subjected to forces Q_1 and Q_2 respectively, resulting from pressure, friction, and impact of particles. The physical properties of both phases are assumed constant over the specific channel segment, the impact of dust particles on the plate is assumed inelastic, and the stream is considered steady and incompressible. In order to account for the two-dimensionality of the flow here, the stream is divided into two regions in each of which the flow is assumed one-dimensional: in one region (width H_0' far in front of the expeller) the dust particles collide with the plate, in the other region they do not collide. The air-velocity field is plotted on the basis of potential-flow calculations, whereupon the trajectories of dust particles are found by integrating the equation of motion for single spherical such particles. From the equations of mass, momentum, and energy conservation (with losses during particle acceleration and deceleration accounted for) at sections 0-0, 1-1, and 2-2, one obtains analytical expressions for Q_1, Q_2, and $Q_2/Q_1 = \Pi(\mu, \delta)$, with u denoting the weight concentration, δ denoting the dimension of a single particle,
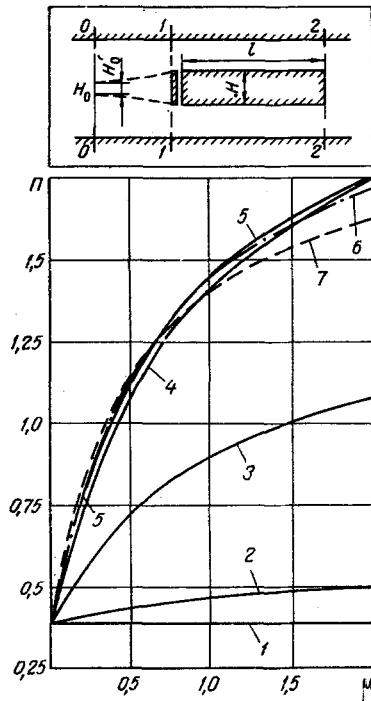

Fig. 1. Ratio of forces Q_2/Q_1
= Π as a function of the dust
concentration μ: $\delta \to 0$ (1),
$\delta = 5\,\mu m$ (2), $\delta = 10\,\mu m$ (3),
$\delta = 15\,\mu m$ (4), $\delta = 20\,\mu m$ (5),
$\delta = 30$ m (6), $\delta \to \infty$ (7).

$$\Pi = \cfrac{2\mu\,(c_2 - \gamma c_1) + (1 + \mu)\,\cfrac{l}{H_0 - H}\,\zeta}{1 - \sigma + \mu\left(c_1\gamma + c_1'\,\sigma\cfrac{1-\gamma}{1-\sigma} - c_0\sigma\cfrac{\gamma-\sigma}{1-\sigma}\right)},$$

c denoting the ratio of dust velocity to air velocity, subscripts 0, 1, 2 referring to the respective sections shown on the diagram, prime sign (') referring to the dust jet whose particles collide with the expeller, $\gamma = 1 - H_0'/H_0$, $\sigma = 1 - H/H_0$, and ζ denoting the friction coefficient. Quantities c_1, c_2, c_1', and γ are functions of the particle dimension.

Calculations have been made for 5, 10, 15, 20, and 30 μm particles and Π-characteristics of this dust flow meter have been plotted (Fig. 1), for application to carbon dust measurements in dusts feeding the burners of boilers aggregates, according to which concentration measurements based on determining the parameter Π are sufficiently sensitive up to $\mu = 1$. One result of practical importance is that Π does not depend much on the particle dimensions, as long as $\delta < 15\,\mu m$, which has been revealed in tests with throttle-type dust flow meters [1, 2] but has not yet been properly explained. The values of forces Q_1 and Q_2 as well as of the pressure drops obtained for this type of dust flow meter on the basis of a two-dimensional flow differ appreciably from the values obtained by the conventional one-dimensional method.

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EVALUATING THE ACCURACY OF GAMMA-RAY MEASUREMENTS IN SEDIMENTARY DEPOSITS*

Ya. P. Boikova and N. P. Glazkov

UDC 627.157

New ways to improve the accuracy of gamma-ray measurements with a transmittance geometry in sedimentary deposits are explored. The effective cross sections of sandstone molecules (sandstone is the principal component of sedimentary deposits) are estimated, for this purpose, in terms of gamma-ray attenuation due to Compton scattering and due to photoelectric absorption within the 20-1300 keV energy band. On the basis of calculations, the relative attenuation is plotted as a function of sediment concentration, along with the linear attenuation factor and the measurement error as functions of the gamma-ray energy. From these graphs one may conclude that the sensitivity and the accuracy of sediment measurements by the gamma-ray method can be improved 50 times to 0.1 g/liter, if a soft 22 keV cadmium-109 source is used instead of a 1300 keV cobalt-60 source.

TEMPERATURE DISTRIBUTION IN A CYLINDER WITH A MOVING HEAT SOURCE†

R. I. Blyakhman

UDC 536.21

The transient temperature field in a semiinfinite circular cylinder is determined analytically for the case where a point source moves at constant velocity along a chord of the end section. The source output, which varies arbitrarily with time, is incorporated into the specific power of heat sources in terms of improper Dirac functions. The thermophysical properties are considered constant. It is assumed, furthermore, that the heat transfer between the cylindrical surface and the ambient medium follows Newton's law and that heat dissipation from the end surface is negligible.

The problem is solved with the aid of Fourier–Hankel integral transformations.

The solution, obtained in universal coordinates and extended to a cylinder of finite length, is suitable for computer-aided calculations of temperature field during material processing operations.

APPLICATION OF THE FOURPOLE NETWORK THEORY
TO AN ANALYSIS OF TEMPERATURE FIELDS IN
COMPOSITE BODIES

L. A. Savintseva, E. D. Strelova,
and N. N. Tarnovskii
UDC 536.2

A procedure is outlined for numerically analyzing the heat transmission through a composite body in whose component parts the temperature fields are either uniform or one-dimensional, with heat being transferred from one generating surface to another and to the ambient medium. The theoretical model, where the properties of the body and the processes in the body are idealized, is treated as a thermal network. The latter is represented by an equivalent circuit diagram with lumped thermal impedances, thermal current sources, and temperature-difference sources. The thermal circuit is calculated on the basis of Ohm's law and Kirchhoff's laws, allowing a determination of mean temperatures at the surfaces of the body elements and at the interfaces between the elements. The interface temperatures can then be used for determining the temperature distribution in the body elements.

The circuit representation of elements (bodies) with one-dimensional temperature fields and with heat dissipation at the generating surfaces is based on the electric fourpole network theory.

A common form of elements in a composite body, namely a disc of uniform thickness with a center hole, is now analyzed. The disc is surrounded by a medium and by bodies at given temperatures. These are all replaced by a fictitious medium whose temperature is equal to the weighted average of all given temperatures. The heat transfer processes in such a system are represented by three thermal conductances forming a T-network and by a source of temperature difference. The latter corresponds to the given temperature of the fictitious medium.

The analytical expressions for the conductances in the circuit are derived from the exact solution to the equation of heat conduction*

$$Y_0 = 2\pi r_1^2 (\alpha_{\text{B}} + \alpha_{\text{H}}) [K_1(m_1) I_1(m_2) - I_1(m_1) K_1(m_2)],$$

$$Y_1 = Y_0 / \{m_2 [I_0(m_1) K_1(m_2) + K_0(m_1) I_1(m_2)] - 1\},$$

$$Y_2 = Y_0 / \{m_1 [K_1(m_1) I_0(m_2) + I_1(m_1) K_0(m_2)] - 1\},$$

$$m_1 = r_1 \sqrt{\frac{\alpha_u + \alpha_l}{\lambda\delta}}, \quad m_2 = r_2 \sqrt{\frac{\alpha_u + \alpha_l}{\lambda\delta}},$$

where r_2 and r_1 denote the outside and the inside (hole) radius of the disc respectively; I_0, I_1, K_0, K_1 are modified zeroth-order and first-order Bessel and MacDonald functions respectively.

The proposed method is further illustrated in the calculation of the temperature field in a system composed of a rod and a circular disc.

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*The thermal conductivity of the disc material λ and the coefficients of heat transfer from the upper and from the lower flat surface to the fictitious medium d_u and d_l are functions of neither the coordinates nor the temperature.

DETERMINING THE ICE CONTENT IN ROCKS OF A MASSIF

A. V. Rashkin, N. G. Shuvalov,
and Yu. M. Vedyaev

A deficiency of both laboratory and field methods of determining the ice content in rocks is that the rock structure becomes distorted during tests and that the latter are cumbersome in view of the need for costly drilling operations. The method proposed here, by which the ice content is determined from the amount of energy drawn from a line source for phase transformation within a cylindrical massif between two or several parallel crevices, will eliminate this deficiency.

A test was performed in one of the districts permanently frozen with a seasonal 2 m deep thaw layer. Thermocouples were installed in two crevices (10 m deep and 0.3 m apart) and their emfs were recorded by means of a model GZP-42 mirror galvanometer with a model PP-63 potentiometer. In one crevice was also installed a linear electric heater with a power $P_N = 93$ W per running meter.

The ice content i was calculated from the equation of heat balance

$$i = \frac{0.86 P_N \tau - Q_n}{79.6 V} \, \text{kg/m}^3, \qquad (1)$$

with the heat Q_n, which is expended on heating the massif within the radius $r_{\lim} = r\sqrt{\tau/\tau_1}$ of the temperature field induced by the heat source, calculated from the temperature change Δt at a distance r from the heater [1]:

$$\Delta t = \frac{0.86 P_N \tau}{2\pi\lambda} \int_{\frac{r}{2\sqrt{ar}}}^{\infty} \frac{1}{x} e^{-x^2} dx, \qquad (2)$$

Here V denotes the volume of thawed massif per unit crevice length (m³/l. m.), m denotes the radius of the thaw zone, τ denotes the thawing time, i.e., the period of heater action (h), and τ_1 denotes the lag time of the temperature wave corresponding to $r_1 : r = 0.3$.

The values of thermal conductivity λ and thermal diffusivity α were selected in accordance with the massif lithology.

A comparison between results of ice content determinations and data obtained by prospecting has demonstrated the applicability of the proposed method to geocryological field studies. An engineering version of this method will facilitate field determinations of thermal characteristics by means of a constant-power cylindrical probe or by means of pulse heating under the exclusion of phase transformations.

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SYMMETRY OF FUNDAMENTAL THERMODYNAMIC QUANTITIES AND RELATIONS

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The symmetry of fundamental thermodynamic quantities and relations between them is revealed and analyzed as a special case of symmetry apparent in various branches of the natural sciences [1]. Such a symmetry can serve as a tool for discovering new quantities and relations as well as for simplifying the derivation and the presentation of numerous thermodynamic equations, especially differential ones [2].

A total of 16 quantities and 24 relations (definitions, equations of the First law, and equations of the Second law of thermodynamics) are classified into 10 kinds (of two types: A and B) of 4 elements each. Quantities A' include volume V, entropy S, pressure P, and temperature T; quantities A" include latent energy (PV) and bonded energy (TS), each with a "+" and a "−" sign; quantities A'" include work, heat, available work [3], and free heat [2]; quantities B" include internal energy, enthalpy, free enthalpy, and free energy. Type-A equations relate quantities A"A', A"A", A"B", and A'"A'; type-B equations relate quantities B"A'" and B"A'.

Four elements in each kind are arranged and consecutive numbers are assigned to them: 1, 3, 5, 7 to type-A quantities and 2, 4, 6, 8 to type-B quantities. The elements with the same index number form a figure; the result are configurations of 8 figures (type A) with 7 elements each and 4 figures (type B) with 3 elements each (shown in Table 1).

The symmetry consists in an invariance of configurations with respect to a group of 8 transformations [1, p. 33] mapping one figure into another of the same type with the kind of element retained.

Since a figure consists of quantities as well as relations, hence a change of quantities according to some rule (code) by a certain transformation yields an automatic mapping of the relations and equations derived from them, which substantiates the method proposed by the author for transforming the equations of thermodynamics [2].

In addition to the null transformation, by which each figure is mapped into itself (with a possible change of sign at the quantities of the pairs P, V and T, S), there are three basic kinds of transformations (I, II, and III) corresponding to the analogies: between single and repetitive processes (I), between adiabatic and isothermal processes (II), between mechanical and thermal quantities (III). The last of these analogies is of a purely formal character, and extending it to the role of symmetric quantities would lead to substantial errors. The remaining transformations represent combinations (successive applications) of the foregoing ones: $II \times I = I \times II$, $III \times I = II \times III$, $III \times II = I \times III$, $III \times II \times I = I \times II \times III$. Distinctive is the symmetry of I, which encompasses not only the fundamental relations but also the equations of state for an ideal gas, the polytropic equations and, specifically, the adiabatic equation.

Table 2 lists the numbers of figures into which figures 1-8 are mapped by each of the 8 transformations. Table 3 shows the combinations and also that any of them results only in transformations among those 8, which, therefore, form a group.

The symmetry in thermodynamics is of the same form as the geometrical symmetry in a fourth-order plane with 8 transformations: 4 reflections and 4 rotations. Table 4 shows the thermodynamic configurations symbolically, as a star of 8 rays 45° apart with odd (type A) and even (type B) ones alternating, also the correspondence between the 8 thermodynamic and the 8 geometrical transformations.

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